

# Electromagnetic Probes of Hot and Dense Hadronic Matter

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## Abstract

The formalism of photon and dilepton productions in a thermal bath are discussed in the frame work of finite temperature field theory. The emission rate has been expressed in terms of the discontinuities or imaginary part of the photon self energy in the thermal bath. The photon and dilepton emissions have been computed by taking into account the in-medium modifications of the particles appearing in the internal loop of the self energy diagram using a phenomenological effective lagrangian approach. The results has been compared with the photon spectra measured by WA 98 collaboration at CERN SPS energies.

## I. Introduction

Numerical simulations of the QCD (Quantum Chromodynamics) equation of state on the lattice predict that at very high density and/or temperature hadronic matter undergoes a phase transition to Quark Gluon Plasma (QGP) [1](see Fig. 1). One expects that ultrarelativistic heavy ion collisions might create conditions conducive for the formation and study of QGP. Various model calculations have been performed to look for observable signatures of this state of matter. However, among various signatures of QGP, photons and dileptons are known to be advantageous, primarily so because, electromagnetic interaction could lead to detectable

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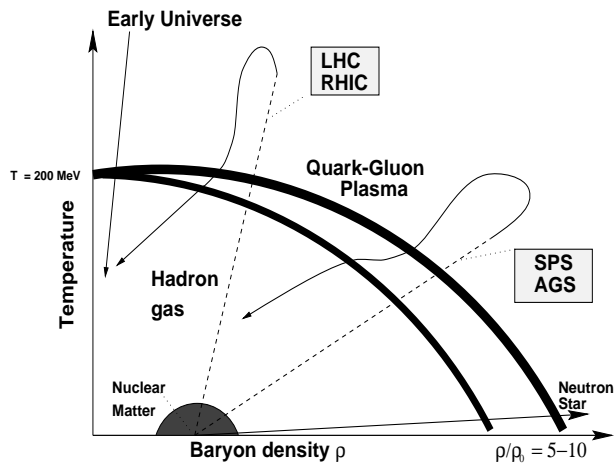


Figure 1: QCD phase diagram.

signal. However, it is weak enough to let the produced particles (real photons and dileptons) escape the system without further interaction and thus carrying the information of the constituents and their momentum distribution in the thermal bath.

The disadvantage with photons is the substantial background from various processes (thermal and non-thermal) [2]. Among these, the contribution from hard QCD processes is well understood in the framework of perturbative QCD and the yield from hadronic decays e. g.  $\pi^0 \rightarrow \gamma\gamma$  can be accounted for by invariant mass analysis. However, photons from the thermalised hadronic gas pose a more difficult task to disentangle. Therefore it is very important to estimate photons from hot and dense hadronic gas along with the possible modification of the hadronic properties.

The importance of the electromagnetic probes for the thermodynamic state of the evolving matter was first proposed by Feinberg in 1976 [3]. While for most purposes one can calculate the emission rates in a classical framework, Feinberg showed that the emission rates can be related to the electromagnetic current current correlation functions in a thermalised system in a quantum picture and, more importantly, in a nonperturbative manner. Generally the production of a particle which interact weakly with the constituents of the thermal bath (the constituents may interact strongly among themselves, we need not give any explicit form of their

coupling strength) can always be expressed in terms of the discontinuities or imaginary parts of the self energies of that particle [4, 5] which never thermalizes. We should, therefore look at the connection between the electromagnetic emission rates (real photons and lepton pairs, which correspond to virtual photons) and the photon spectral function ( which is connected with the discontinuities in self energies) in a thermal system [6], which in turn connected to the hadronic electromagnetic current current correlation tensor [7] through Maxwell equations. We will show below that the photon emission rate can be obtained from the dilepton (which originate from a virtual photon) emission rate with little effort.

In section II we present the general formalism for the dilepton and photon production rate from a thermal bath. In section III we calculate the medium modifications of hadrons. Section IV is devoted to discuss the results of our calculations. In section V we present a summary and discussion.

## II. Electromagnetic Probes - Formulation

We begin our discussion with the dilepton production rate. In the following we will follow the work of Weldon [6] for the derivation of dilepton emission rate. Let  $e_0 J_\mu A^\mu$  be the interaction between the photon and the particles in the heat bath, then to lowest order in the electromagnetic coupling, the  $S$  matrix element for the transition  $| I \rangle \rightarrow | H; l^+ l^- \rangle$  is given by

$$S_{HI} = e_0 \int \langle H; l^+ l^- | J_\mu A^\mu(x) | I \rangle d^4x e^{iq \cdot x} \quad (1)$$

where  $| I \rangle$  is the initial state corresponding to the two incoming nuclei,  $| H; l^+ l^- \rangle$  is the final state which corresponds to a lepton pair plus anything (Hadronic), the parameter  $e_0$  is the bare (unrenormalised) charge and  $q = p_1 + p_2$  is the four momentum of the lepton pair. Since we assume that the lepton pair do not interact with the emitting system, the matrix element can be factorised in the following way,

$$\langle H; l^+ l^- | J_\mu A^\mu(x) | I \rangle = \langle H | A^\mu(x) | I \rangle \langle l^+ l^- | J_\mu | 0 \rangle \quad (2)$$

Putting the explicit form of the current  $J^\mu$  in terms of the Dirac spinors we obtain

$$S_{HI} = e_0 \frac{\bar{u}(p_1) \gamma_\mu v(p_2)}{V \sqrt{2E_1 2E_2}} \int d^4x e^{iq \cdot x} \langle H | A^\mu(x) | I \rangle \quad (3)$$

Therefore dilepton multiplicity from a thermal system is obtained as, (by summing over the final states and averaging over the initial states with a weight factor  $Z(\beta)^{-1} e^{-\beta E_I}$ )

$$N = \frac{1}{Z(\beta)} \sum_I \sum_H |S_{HI}|^2 e^{-\beta E_I} \quad (4)$$

where  $E_I$  is the total energy in the initial state. After some algebra  $N$  can be written in a compact form as follows,

$$N = e_0^2 L^{\mu\nu} H_{\mu\nu} \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} \quad (5)$$

where  $L_{\mu\nu}$  is the leptonic tensor given by

$$\begin{aligned} L^{\mu\nu} &= \frac{1}{4} \sum_{spins} \bar{u}(p_1) \gamma^\mu v(p_2) \bar{v}(p_2) \gamma^\nu u(p_1) \\ &= p_1^\mu p_2^\nu + p_2^\mu p_1^\nu - \frac{q^2}{2} g^{\mu\nu} \end{aligned} \quad (6)$$

and  $H_{\mu\nu}$  is the photon tensor

$$H_{\mu\nu} = \frac{1}{Z(\beta)} e^{-\beta q_0} \sum_H \int d^4x d^4y e^{iq \cdot (x-y)} \langle H | A_\mu(x) A_\nu(y) | H \rangle e^{-\beta E_H} \quad (7)$$

to obtain the above equation we have used the resolution of identity  $1 = \sum_I |I\rangle \langle I|$ , and the energy conservation equation  $E_I = E_H + q_0$  where  $q_0$  is the energy of the lepton pair and  $E_H$  is the energy of the rest of the system produced after collision. Using the translational invariance of the matrix element we can write

$$H_{\mu\nu} = 2\pi \Omega e^{-\beta q_0} \rho^{\mu\nu} \quad (8)$$

where  $\Omega$  is the four volume of the system and  $\rho^{\mu\nu}$  is the spectral function of the photon in the heat bath,

$$\rho_{\mu\nu}(\vec{q}, q_0) = \frac{1}{2\pi Z(\beta)} \int d^4x e^{iq \cdot x} \sum_H \langle H | A_\mu(x) A_\nu(0) | H \rangle e^{-\beta E_H} \quad (9)$$

The rate of dilepton production per unit volume ( $N/\Omega$ ) is given by,

$$\frac{dN}{d^4x} = 2\pi e_0^2 L_{\mu\nu} e^{-\beta q_0} \rho^{\mu\nu}(q) \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} \quad (10)$$

This result which expresses the dilepton emission rate in terms of the spectral function of the photon in the medium is a fundamental result. At zero temperature ( $\beta \rightarrow \infty$ ) the only state which enters in the spectral function ( $\rho(s)$ ) is the vacuum and the spectral function can be expressed in terms of the imaginary part of Greens function (i.e the discontinuity along the real axis)  $\rho(s) = -\frac{1}{\pi} \text{Im } G(s)$  [8].

Unlike at zero temperature, in the case of non-zero temperature the time ordered ( $D^{\mu\nu}$ ) and the Feynman propagator ( $D_F^{\mu\nu}$ ) are different. The relation between them is given by [9],

$$\text{Im } D^{\mu\nu} = (1 + 2f_{BE}) \text{Im } D_F^{\mu\nu} \quad (11)$$

where  $f_{BE}$  is the thermal distribution for photons. Now the time ordered Green's function can be expressed in terms of the spectral function as follows

$$D^{\mu\nu}(q_0, \vec{q}) = \int_{-\infty}^{\infty} d\alpha \left[ \frac{\rho^{\mu\nu}(\alpha, \vec{q})}{q^0 - \alpha + i\epsilon} - \frac{\rho^{\nu\mu}(\alpha, -\vec{q})}{q^0 + \alpha - i\epsilon} \right] \quad (12)$$

Using the KMS relation one can show that

$$\rho^{\mu\nu}(q_0, \vec{q}) = e^{\beta q_0} \rho^{\nu\mu}(-q_0, -\vec{q}) \quad (13)$$

substituting Eq. (13) in Eq. (12) and using Eq. (11) we obtain

$$\rho^{\mu\nu}(q_0, \vec{q}) = \frac{-1}{\pi} (1 + f_{BE}(q_0)) \text{Im } D_F^{\mu\nu}(q_0, \vec{q}) \quad (14)$$

The above equation implies that to evaluate the spectral function at  $T \neq 0$  we need to know the imaginary part of the Feynman propagator. It is important to note that above expression for spectral function reduces its vacuum value as  $\beta \rightarrow \infty$ . The Feynman propagator can be expressed in terms of the proper self energy as

$$D_F^{\mu\nu} = - \left[ \frac{Z_3 A^{\mu\nu}}{q^2 - \Pi_T} + \frac{Z_3 B^{\mu\nu}}{q^2 - \Pi_L} \right] + (\zeta - 1) \frac{q_\mu q_\nu}{q^4} \quad (15)$$

where  $Z_3$  is the wave function renormalization constant at zero temperature,  $A^{\mu\nu}$  and  $B^{\mu\nu}$  are transverse and longitudinal projection tensors respectively. The presence of the parameter  $\zeta$  indicates the gauge dependence of the propagator. Although the gauge dependence cancels out in the calculation of physical quantities, one should, however, be careful when extracting physical quantities from the propagator directly, especially in the non-abelian gauge theory.

Using Eqs. (10), (14) and (15) we get

$$\frac{dN}{d^4x} = 2 e^2 L_{\mu\nu} (A^{\mu\nu} \rho_T + B^{\mu\nu} \rho_L) \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} f_{BE}(q_0) \quad (16)$$

where  $e^2 = Z_3 e_0^2$ , is the physical charge of the electron and

$$\rho_k = \frac{\text{Im } \Pi_k}{(q^2 - \text{Re } \Pi)^2 + \text{Im } \Pi_k^2} \quad (17)$$

The emission rate of dilepton can also be expressed in terms of the electromagnetic current-current correlation functions [7]. Denoting the hadronic part of the electromagnetic current operator by  $J_\mu^h$ , the leptonic part by  $J_\nu^l$  and the free photon propagator by  $D^{\mu\nu}$ , we have the matrix element for this transition :

$$S_{HI} = \langle H; l^+ l^- \mid \int d^4x d^4y J_\mu^h(x) D^{\mu\nu} J_\nu^l(y) \mid I \rangle \quad (18)$$

As in the earlier case the leptonic part of the current can be easily factored out. Writing the Fourier transform of photon propagator and squaring the matrix elements, one obtains, for the rate of dilepton production,

$$dR = e_0^2 L^{\mu\nu} W_{\mu\nu} \frac{1}{q^4} \frac{d^3 p_1}{(2\pi)^3 E_1} \frac{d^3 p_2}{(2\pi)^3 E_2} \quad (19)$$

where  $W^{\mu\nu}(q)$  is just the Fourier transform of the thermal expectation value of the real time electromagnetic current-current correlation function :

$$W^{\mu\nu} = \int d^4x e^{-iqx} \sum_H \langle H \mid J^\mu(x) J^\nu(0) \mid H \rangle \frac{e^{-\beta E_H}}{Z} \quad (20)$$

The subtleties of the thermal averaging have been elucidated earlier. It is thus readily seen (eq. 19) that the dilepton data yields considerable

information about the thermal state of the hadronic system; at least the full tensor structure of  $W^{\mu\nu}$  can in principle be determined. The most important point to realize here is that the analysis is essentially nonperturbative.

At this point one may wonder what is the connection between the electromagnetic current-current correlation function and the spectral function? The connection can be expressed in a straight forward way by substituting  $J_\mu$  and  $J_\nu$  using Maxwell equation  $(\partial_\alpha \partial^\alpha A^\mu - \partial^\mu (\partial_\alpha A^\alpha) = j^\mu)$  in Eq. (20) to obtain,

$$e_0^2 W^{\mu\nu} = 2\pi (q^2 g^{\mu\alpha} - q^\mu q^\alpha) \rho_{\alpha\beta} (q^2 g^{\beta\nu} - q^\beta q^\nu) \quad (21)$$

In the literature most of the dilepton production rate from a thermal system are calculated with the approximation  $\Pi_T = \Pi_L$  which gives the spectral function as

$$\rho^{\mu\nu}(q_0, \vec{q}) = (-g^{\mu\nu} + q^\mu q^\nu / q^2) \frac{-1}{\pi} (1 + f_{BE}(q_0)) \text{Im} \left[ \frac{1}{q^2 - \Pi_L} \right] \quad (22)$$

Since  $\text{Im} \Pi_k$  and  $\text{Re} \Pi_k$  are proportional to  $\alpha$  (the fine structure constant) they are very small for most of our practical purposes. Therefore  $\rho_k$  is given by

$$\rho_k = \frac{\text{Im} \Pi_k}{q^4} \quad (23)$$

Using Eqs. (16), (22), (23) and the following result

$$\begin{aligned} \int \prod_{i=1,2} \frac{d^3 p_i}{(2\pi)^3 2E_i} \delta^4(p_1 + p_2 - q) L^{\mu\nu}(p_1, p_2) &= \frac{1}{(2\pi)^6} \frac{2\pi}{3} \left(1 + \frac{2m^2}{q^2}\right) \\ &\times \sqrt{1 - \frac{4m^2}{q^2}} C^{\mu\nu} \quad (24) \end{aligned}$$

where  $C^{\mu\nu} = q^\mu q^\nu - q^2 g^{\mu\nu}$ , we get,

$$\frac{dR}{d^4 q} = \frac{\alpha}{12\pi^4 q^2} \left(1 + \frac{2m^2}{q^2}\right) \sqrt{1 - \frac{4m^2}{q^2}} \text{Im} \Pi_\mu^\mu f_{BE}(q_0) \quad (25)$$

This is the familiar results most widely used for dilepton emission rate [5].

To obtain real photon emission rate per unit volume ( $dR$ ) from a system under thermal equilibrium we note that the dilepton emission rate

differs from the photon emission rate in the following way. The factor  $e^2 L_{\mu\nu} \frac{1}{q^4}$  is the product of the electromagnetic vertex  $\gamma^* \rightarrow l^+ l^-$ , the leptonic current involving Dirac spinors and the square of the photon propagator should be replaced by the factor  $\sum \epsilon_\mu \epsilon_\nu (= -g_{\mu\nu})$  for the real (on-shell) photon. Finally the phase space factor  $d^3 p_1 / [(2\pi)^3 2 E_1] d^3 p_2 / [(2\pi)^3 2 E_2]$  should be replaced by,  $d^3 q / [(2\pi)^3 2 q_0]$  to obtain

$$dR = -\frac{1}{(2\pi)^3} g_{\mu\nu} W^{\mu\nu} \frac{d^3 q}{q_0} \quad (26)$$

The current-current correlation function is related to the photon self energy as [5, 10, 11]

$$W_{\mu\nu} = 2 f_{BE}(q_0) \text{Im} \Pi_{\mu\nu} \quad (27)$$

Therefore,

$$q_0 \frac{dR}{d^3 q} = -2 \frac{1}{(2\pi)^3} \text{Im} \Pi_\mu^\mu f_{BE}(q_0) \quad (28)$$

The above equation can also be obtained directly from eq. 25. The emission rate given above is correct up to order  $e^2$  in electromagnetic interaction but exact, in principle, to all order in strong interaction. But for all practical purposes one is able to evaluate up to a finite order of loop expansion. Now it is clear from the above results that to evaluate photon and dilepton emission rate from a thermal system we need to evaluate the imaginary part of the photon self energy. The Cutkosky rules at finite temperature [12, 13, 14] gives a systematic procedure to calculate the imaginary part of a Feynman diagram. The Cutkosky rule expresses the imaginary part of the  $n$ -loop amplitude in terms of physical amplitude of lower order ( $n - 1$  loop or lower). This is shown schematically in Fig. 2. When the imaginary part of the self energy is calculated up to and including  $L$  order loops where  $L$  satisfies  $x + y < L + 1$  then we obtain the photon emission rate for the reaction;  $x$  particles  $\rightarrow y$  particles  $+\gamma$  and the above formalism becomes equivalent to the relativistic kinetic theory formalism [15].

For a reaction  $1 + 2 \rightarrow 3 + \gamma$  the photon (of energy  $E$ ) emission rate is given by [16],

$$E \frac{dR}{d^3 p} = \frac{N}{16(2\pi)^7 E} \int_{(m_1+m_2)^2}^{\infty} ds \int_{t_{\min}}^{t_{\max}} dt |\mathcal{M}|^2 \int dE_1$$



$$\text{Im}\left(\text{Diagram with two incoming } \gamma \text{ and two outgoing } \gamma\right) = \text{Im}\left(\text{Diagram 1} + \text{Diagram 2} + \dots\right)$$

$$= \left| \text{Diagram 3} \right|^2$$

Figure 2: Optical Theorem in Quantum Field Theory

$$\times \int dE_2 \frac{f(E_1) f(E_2) [1 + f(E_3)]}{\sqrt{aE_2^2 + 2bE_2 + c}} \quad (29)$$

where

$$\begin{aligned} a &= -(s + t - m_2^2 - m_3^2)^2 \\ b &= E_1(s + t - m_2^2 - m_3^2)(m_2^2 - t) + E[(s + t - m_2^2 - m_3^2)(s - m_1^2 - m_2^2) \\ &\quad - 2m_1^2(m_2^2 - t)] \\ c &= -E_1^2(m_2^2 - t)^2 - 2E_1E[2m_2^2(s + t - m_2^2 - m_3^2) - (m_2^2 - t)(s - m_1^2 - m_2^2)] \\ &\quad - E^2[(s - m_1^2 - m_2^2)^2 - 4m_1^2m_2^2] - (s + t - m_2^2 - m_3^2)(m_2^2 - t) \\ &\quad \times (s - m_1^2 - m_2^2) + m_2^2(s + t - m_2^2 - m_3^2)^2 + m_1^2(m_2^2 - t)^2 \\ E_{1\min} &= \frac{(s + t - m_2^2 - m_3^2)}{4E} + \frac{Em_1^2}{s + t - m_2^2 - m_3^2} \\ E_{2\min} &= \frac{Em_2^2}{m_2^2 - t} + \frac{m_2^2 - t}{4E} \\ E_{2\max} &= -\frac{b}{a} + \frac{\sqrt{b^2 - ac}}{a} \end{aligned}$$

In a similar way the dilepton emission rate for a reaction  $a \bar{a} \rightarrow l^+ l^-$  can be obtained as

$$\frac{dN}{d^4x d^3q dM} = \int \frac{d^3p_a}{2E_a(2\pi)^3} f(p_a) \int \frac{d^3p_{\bar{a}}}{2E_{\bar{a}}(2\pi)^3} f(p_{\bar{a}}) \int \frac{d^3p_1}{2E_1(2\pi)^3} \int \frac{d^3p_2}{2E_2(2\pi)^3}$$

$$|M|_{a\bar{a} \rightarrow l+l-}^2 = (2\pi)^4 \delta(\vec{p}_a + \vec{p}_{\bar{a}} - \vec{p}_1 - \vec{p}_2) \delta(E_a + E_{\bar{a}} - E_1 - E_2) \delta(p_a + p_{\bar{a}}) \delta(M - E_a - E_{\bar{a}}) \quad (30)$$

where  $M^2 = (p_a + p_{\bar{a}})^2$  is the invariant mass and  $f(p_a)$  is the occupation probability in the momentum space. The Pauli blocking of the lepton pair in the final state has been neglected in the above equation.

### III. Finite Temperature Properties

To calculate the effective mass and decay widths of vector mesons we begin with the following  $VNN$  (Vector - Nucleon - Nucleon) Lagrangian density:

$$\mathcal{L}_{VNN} = g_{VNN} \left( \bar{N} \gamma_\mu \tau^a N V_a^\mu - \frac{\kappa_V}{2M} \bar{N} \sigma_{\mu\nu} \tau^a N \partial^\nu V_a^\mu \right), \quad (31)$$

where  $V_a^\mu = \{\omega^\mu, \vec{\rho}^\mu\}$ ,  $M$  is the free nucleon mass,  $N$  is the nucleon field and  $\tau_a = \{1, \vec{\tau}\}$ . The values of the coupling constants  $g_{VNN}$  and  $\kappa_V$  will be specified later. With the above Lagrangian we proceed to calculate the  $\rho$ -self energy.

$$\Pi_{\mu\nu} = -2ig_{VNN}^2 \int \frac{d^4p}{(2\pi)^4} K_{\mu\nu}(p, k), \quad (32)$$

with,

$$K_{\mu\nu} = \frac{\text{Tr} [\Gamma_\mu(k) (\not{p} + M^*) \Gamma_\nu(-k) (\not{p} - \not{k} + M^*)]}{(p^2 - M^{*2})((p - k)^2 - M^{*2})}, \quad (33)$$

The vertex  $\Gamma_\mu(k)$  is calculated by using the Lagrangian of Eq. (31) and is given by

$$\Gamma_\mu(k) = \gamma_\mu + \frac{i\kappa_V}{2M} \sigma_{\mu\alpha} k^\alpha. \quad (34)$$

Here  $M^*$  is the in-medium (effective) mass of the nucleon at finite temperature which we calculate using the Mean-Field Theory (MFT) [17]. The value of  $M^*$  can be found by solving the following self consistent equation:

$$M^* = M - \frac{4g_s^2}{m_s^2} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{M^*}{(\mathbf{p}^2 + M^{*2})^{1/2}} [f_N(T) + f_{\bar{N}}(T)], \quad (35)$$

where  $f_N(T)(f_{\bar{N}}(T))$  is the Fermi-Dirac distribution for the nucleon (antinucleon),  $m_s$  is mass of the neutral scalar meson ( $\sigma$ ) field, and, the nucleon interacts via the exchange of isoscalar meson with coupling constant  $g_s$ . Since the exact solution of the field equations in QHD is untenable, these are solved in mean field approximation. In a mean field approximation one replaces the field operators by their ground state expectation values which are classical quantities; this renders the field equations exactly solvable.

The vacuum part of the  $\rho$  self energy arises due to its interaction with the nucleons in the Dirac sea. This is calculated using dimensional regularization scheme (see [18] for details).

In a hot system of particles, there is a thermal distribution of real particles (on shell) which can participate in the absorption and emission process in addition to the exchange of virtual particles. The interaction of the rho with the onshell nucleons, present in the thermal bath contributes to the medium dependent part of the  $\rho$ -self energy. This is calculated from Eq.(32) using imaginary time formalism as,

$$\begin{aligned} \text{Re}\Pi_{\text{med}}(\omega, \mathbf{k} \rightarrow 0) = & -\frac{16g_{VNN}^2}{\pi^2} \int \frac{p^2 dp}{\omega_p (e^{\beta\omega_p} + 1)(4\omega_p^2 - \omega^2)} \\ & \times \left[ \frac{1}{3}(2p^2 + 3M^{*2}) + \omega^2 \left\{ M^* \left( \frac{\kappa_V}{2M} \right) \right. \right. \\ & \left. \left. + \frac{1}{3} \left( \frac{\kappa_V}{2M} \right)^2 (p^2 + 3M^{*2}) \right\} \right] \end{aligned} \quad (36)$$

where,  $\omega_p^2 = p^2 + M^{*2}$ .

## IV. Results

In this section we present the results of our calculation of the effective masses and decay widths of vector mesons and its effect on electromagnetic probes. For our calculations of  $\rho$  and  $\omega$  mesons effective masses we have used the following values of the coupling constants and masses [19]:  $\kappa_\rho = 6.1$ ,  $g_{\rho NN}^2/4\pi = 0.55$ ,  $m_s = 550$  MeV,  $m_\rho = 770$  MeV,  $M = 938$  MeV,  $g_s^2/4\pi = 9.3$ ,  $g_{\omega NN}^2/4\pi = 20$ , and  $\kappa_\omega = 0$ . In Fig. 3 we show the variation of the ratio of effective mass to free mass for hadrons as a function of temperature. The expected trend based on Brown - Rho scaling

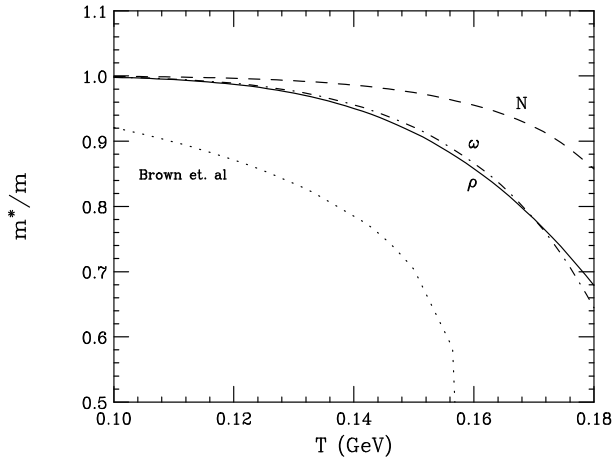


Figure 3: Ratio of effective mass to free space mass of hadrons as a function of temperature  $T$ .

is also shown. Firstly, we do not observe any global scaling behaviour. Secondly, in our case the effective mass as a function of temperature falls at a slower rate.

Many authors have investigated the issue of temperature dependence of hadronic masses within different models over the past several years. Hatsuda and collaborators [20, 21, 22] and Brown [23] showed that the use of QCD sum rules at finite temperature results in a temperature dependence of the  $q\bar{q}$  condensate culminating in the following behaviour of the  $\rho$ -mass:

$$\frac{m_\rho^*}{m_\rho} = \left(1 - \frac{T^2}{T_\chi^2}\right)^{1/6} \quad (37)$$

where  $T_\chi$  is the critical temperature for chiral phase transition. Brown and Rho [24] also showed that the requirement of chiral symmetry yields an approximate scaling relation between various effective hadron masses,

$$\frac{M^*}{M} \approx \frac{m_\rho^*}{m_\rho} \approx \frac{m_\omega^*}{m_\omega} \approx \frac{f_\pi^*}{f_\pi} \quad (38)$$

These calculations show a dropping of hadronic mass with temperature. Calculations with non-linear  $\sigma$ -model, however, predict the opposite trend [25].

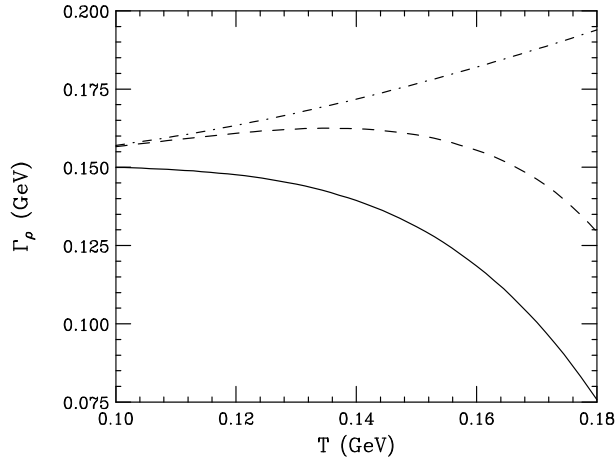


Figure 4: Rho decay width ( $\Gamma_\rho$ ) as a function of temperature ( $T$ ). Dashed and solid lines show calculations of rho width with effective mass due to nucleon loop, with and without BE. Dot dashed line represents the same but with effective mass due to pion loop.

The variation of the in-medium decay width ( $\Gamma_\rho$ ) of the rho meson with temperature is shown in Fig. (4). As discussed earlier, the effective mass of the rho decreases as a result of  $N\bar{N}$  polarisation. This reduces the phase space available for the rho. Hence, we observe a rapid decrease in the rho meson width with temperature (solid line). However, the presence of pions in the medium would cause an enhancement of the decay width through induced emission. Thus when Bose Enhancement (BE) of the pions is taken into account the rho decay width is seen to fall less rapidly (dashed line); such a behaviour is observed quite clearly in [26]. For the sake of completeness, we also show the variation of rho width in the case where the rho mass changes due to  $\pi\pi$  loop. In this case, since the rho mass increases (though only marginally), the width increases (dot-dashed line).

Now we present our results on photon emission rates from a hot hadronic gas. As discussed earlier, the variation of hadronic decay widths and masses will affect the photon spectra. The relevant reactions of photon production are  $\pi\pi \rightarrow \rho\gamma$ ,  $\pi\rho \rightarrow \pi\gamma$ ,  $\pi\pi \rightarrow \eta\gamma$ , and  $\pi\eta \rightarrow \eta\gamma$  with all possible isospin combinations. Since the lifetimes of the rho and omega mesons are comparable to the strong interaction time scales, the

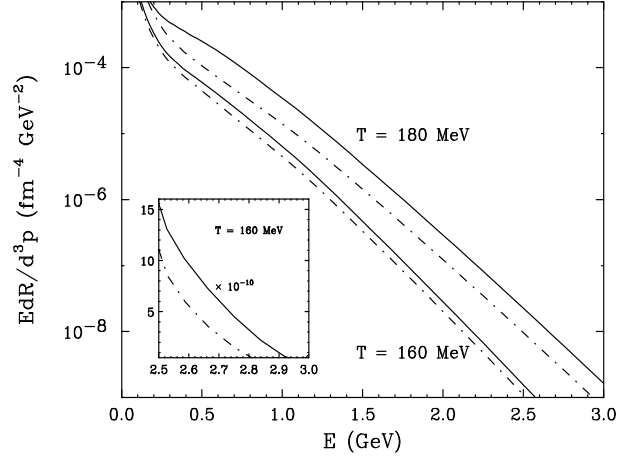


Figure 5: Total photon emission rate from hot hadronic matter as a function of photon energy at  $T=160, 180$  MeV. The solid and dot-dashed lines show results with and without in-medium effects respectively. Inset: Total photon emission rate is plotted in linear scale as a function of photon energy in the kinematic window,  $E_\gamma = 2.5$  to  $3.0$  GeV at  $T = 180$  MeV.

decays  $\rho \rightarrow \pi\pi\gamma$  and  $\omega \rightarrow \pi^0\gamma$  are also included. The effect of finite resonance width of the rho meson in the photon production cross-sections has been taken into account through the propagator.

In Fig. (5) we display the total rate of emission of photons from hot hadronic gas including all hadronic reactions and decays of vector mesons. At  $T = 180$  MeV, the photon emission rate with finite temperature effects is a factor of  $\sim 3$  higher than the rate calculated without medium effects. At  $T = 160$  MeV the medium effects are small compared to the previous case.

The transverse momentum distribution of photon has been obtained by convoluting the production rate with the space time dynamics of the system. The boost invariant hydrodynamical model of Bjorken [27] has been used for the space time evolution of the system with appropriate modification due to the shift in hadronic masses [28]. In Fig. 6 we compare our results of transverse momentum distribution of photons with the preliminary results of WA98 Collaboration [29]. The experimental data represents the photon spectra from Pb + Pb collisions at 158 GeV per

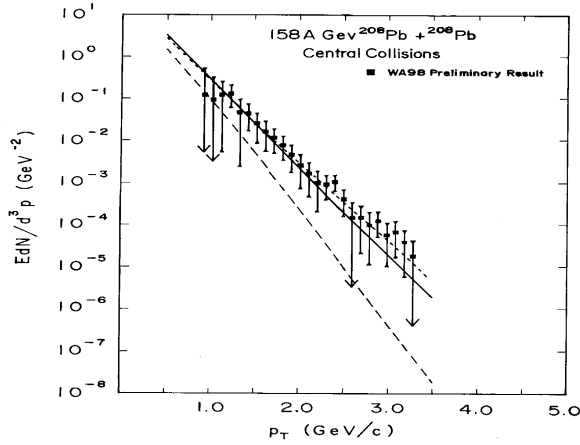


Figure 6: Total thermal photon yield in Pb + Pb central collisions at 158 GeV per nucleon at CERN SPS. The long-dash line shows the results when the system is formed in the QGP phase with initial temperature  $T_i = 180$  MeV at  $\tau_i = 1$  fm/c. The critical temperature for phase transition is taken as 160 MeV. The solid (short-dash) line indicates photon spectra when hadronic matter formed in the initial state at  $T_i = 230$  MeV ( $T_i = 270$  MeV) at  $\tau_i = 1$  fm/c with (without) medium effects on hadronic masses and decay widths.

nucleon at CERN SPS energies. The transverse momentum distribution of photons originating from the ‘hadronic scenario’ (matter formed in the hadronic phase) with (solid line) and without (short-dash line) medium modifications of vector mesons outshine the photons from the ‘QGP scenario’ (matter formed in the QGP phase, indicated by long-dashed line) for the entire range of  $p_T$ . Although photons from ‘hadronic scenario’ with medium effects on vector mesons shine less bright than those from without medium effects for  $p_T > 2$  GeV, it is not possible to distinguish clearly between one or the other on the basis of the experimental data.

So far we have studied the effects of in-medium hadronic masses and decay widths on the photon spectra, where, the latter is found to have negligible effects. However, the modification in the rho decay width due to BE, arising from the induced emission of pions in the thermal medium, plays a crucial role in low mass lepton pair production. It has been shown in Ref. [30] that this effect arises naturally in a calculation based on finite

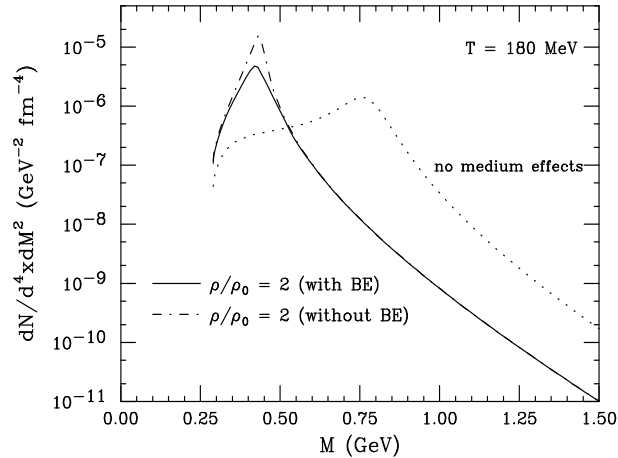


Figure 7: Dilepton emission rates as a function of invariant mass at  $T = 180$  MeV and twice nuclear matter density. Solid and dotdashed lines show the results with and without BE effects respectively. The dotted line shows the result when no medium effect is considered.

temperature field theory.

For the sake of illustration, we consider the effect of thermal nucleon loop on the lepton pair production from pion annihilation via (rho mediated) vector meson dominance. In the work of Li, Ko and Brown [31], the observed enhancement on the low mass dilepton yield was attributed solely to the the matter induced mass modification of the meson, while neglecting the BE effect. In Fig. (7) the dilepton emission rate is plotted as a function of invariant mass at a temperature  $T=180$  MeV. It is clear from the figure that the inclusion of BE effects reduces the yield by increasing the decay width. Quantitatively, a suppression by a factor of 3 is observed at the invariant mass,  $M = m_\rho^*$ .

## V. Summary and Discussions

We have discuss the general formalism to evaluate the emission of electromagnetic ejectiles from a thermal bath. This formalism has been applied to evaluate the photon and dilepton emission rate from a hot and dense hadronic system. We have calculated the effective masses and decay widths of vector mesons propagating in a hot medium. We have



seen that the mass of rho meson decreases substantially due to its interactions with nucleonic excitations and it increases only marginally ( $\sim 10\text{-}15$  MeV) due to  $\rho - \pi$  interactions. The overall decrease in the effective mass is due to fluctuations in the Dirac sea of nucleons with mass  $M^*$  and the in-medium contribution increases very little ( $\sim 5 - 10$  MeV) from its free space value. The omega mass drops at  $T = 180$  MeV by about 250 MeV from its free space value.

We have evaluated the rate of photon emission from a hadronic gas of  $\pi, \rho, \omega$  and  $\eta$  mesons. It is seen that the photon rate increases by a factor  $\sim 3$  due to the inclusion of in-medium masses at  $T = 180$  MeV. We observe that the inclusion of in-medium decay widths in vector meson propagator has negligible effects on photon emission rates, although the effect of in-medium decay widths with BE is substantial for the dilepton yield.

We have compared the experimental data on photon spectra from Pb + Pb collisions at energies 158 GeV per nucleon with different initial conditions. We observe that photons from hadronic scenario dominates over the photons from QGP scenario for the entire  $p_T$  domain. But in the hadronic scenario the photon spectra evaluated with and without in-medium properties of vector mesons describe these data reasonably well. Hence the transverse photon spectra at present do not allow us to decide between an in-medium dropping mass and a free mass scenario. Considering the experimental uncertainty, it is not possible to state, which one, between the two is compatible with the data. Experimental data with better statistics could possibly distinguish among various scenarios.

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